Forma esponenziale	Forma logaritmica
$10^4 = 10.000$	$\log_{10} 10.000 = 4$
$2^5 = 32$	$\log_2 32 = 5$
$3^3 = 27$	$\log_3 27 = 3$
$7^1 = 7$	$\log_7 7 = 1$
$5^0 = 1$	$\log_5 1 = 0$
$3^{-2} = \frac{1}{9}$	$\log_3 \left(\frac{1}{9}\right) = -2$
$8^{1/3} = 2$	$\log_8 2 = \frac{1}{3}$

Proprietà della funzione esponenziale e corrispondenti proprietà del logaritmo naturale

Esponenziale	Logaritmo naturale
$e^{\ln x} = x$	$\ln(e^x) = x$
$e^0 = 1$	$\ln 1 = 0$
$e^1 = e$	$\ln e = 1$
$e^x e^y = e^{x+y}$	$\ln(xy) = \ln x + \ln y$
$(e^x)^y = e^{xy}$	$\ln(x^y) = y \ln x$
$\frac{e^x}{e^y} = e^{x-y}$	$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
$e^{-x} = \frac{1}{e^x} = \left(\frac{1}{e}\right)^x$	

Algebra estesa dei limiti

$(+\infty) + a = +\infty + a = +\infty$ con $a \in \mathbb{R}$

$(-\infty) + a = -\infty + a = -\infty$ con $a \in \mathbb{R}$

$(+\infty) + (+\infty) = +\infty + \infty = +\infty$

$(-\infty) + (-\infty) = -\infty - \infty = -\infty$

$$(+\infty) \cdot a = \begin{cases} +\infty & \text{se } a > 0 \\ -\infty & \text{se } a < 0 \end{cases}$$

$$(-\infty) \cdot a = \begin{cases} -\infty & \text{se } a > 0 \\ +\infty & \text{se } a < 0 \end{cases}$$

$$(\pm\infty) \cdot (\pm\infty) = +\infty$$

$$(\pm\infty) \cdot (\mp\infty) = -\infty$$

$$\frac{1}{0^{\pm}} = \pm\infty \qquad \frac{1}{\pm\infty} = 0^{\pm}$$

$$\frac{a}{0^{\pm}} = a \cdot \frac{1}{0^{\pm}} = a \cdot (\pm\infty) = \begin{cases} \pm\infty & \text{se } a > 0 \\ \mp\infty & \text{se } a < 0 \end{cases}$$

$$\frac{a}{\pm\infty} = a \cdot \frac{1}{\pm\infty} = a \cdot 0^{\pm} = \begin{cases} 0^{\pm} & \text{se } a > 0 \\ 0^{\mp} & \text{se } a < 0 \end{cases}$$

$$\frac{0^{\pm}}{\pm\infty} = 0^{\pm} \cdot \frac{1}{\pm\infty} = 0^{\pm} \cdot 0^{\pm} = 0^{+}$$

$$\frac{0^{\mp}}{\pm\infty} = 0^{\mp} \cdot \frac{1}{\pm\infty} = (0^{\mp}) \cdot (0^{\pm}) = 0^{-}$$

$$\frac{\pm\infty}{0^{\pm}} = \pm\infty \cdot \frac{1}{0^{\pm}} = (\pm\infty)\cdot(\pm\infty) = +\infty$$

$$\frac{\mp\infty}{0^{\pm}} = \mp\infty \cdot \frac{1}{0^{\pm}} = (\mp\infty)\cdot(\pm\infty) = -\infty$$

Forme indeterminate

$$(+\infty) + (-\infty) = +\infty - \infty$$

$$0 \cdot (\pm\infty) = 0 \cdot \infty$$

$$\frac{0^{\pm}}{0^{\pm}} = \frac{0^{\pm}}{0^{\mp}} = \frac{0}{0}$$

$$\frac{\pm\infty}{\pm\infty} = \frac{\pm\infty}{\mp\infty} = \frac{\infty}{\infty}$$

$$1^{\infty} \iff e^{\ln 1^{\infty}} \iff e^{\infty \ln 1} = e^{\infty \cdot 0}$$

$$0^{0} \iff e^{\ln 0^{0}} \iff e^{0 \cdot \ln 0} = e^{0 \cdot \infty}$$

$$\infty^{0} \iff e^{\ln \infty^{0}} \iff e^{0 \cdot \ln \infty} = e^{0 \cdot \infty}$$

Tecniche per risolvere i limiti con forme indeterminate

1. manipolazioni algebriche
2. infinitesimi e infiniti
3. limiti notevoli
4. regola di de L'Hôpital
5. Formula di Taylor - Mac Laurin

Limiti notevoli

- $$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$$

In generale

$$\lim_{x \to \pm\infty} \left(1 + \frac{a}{x}\right)^x = e^a \quad \text{con } a \in \mathbb{R}$$

- $$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

In generale

$$\lim_{x \to 0} \frac{\log_a(1+x)}{x} = \frac{1}{\ln a} \quad \text{con } a > 1$$

- $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

In generale

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \quad \text{con } a > 1$$

- $$\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha \quad \text{con } \alpha \in \mathbb{R}$$

Derivate delle funzioni elementari

$(f(x))$	$(f'(x))$
c	0
x	1
$mx+q$	m
x^{α} $(\alpha \in \mathbb{R})$	$\alpha x^{\alpha-1}$
a^{x} $(a \in \mathbb{R}_{++})$	$a^{x} \ln a$
e^{x}	e^{x}
$\log_a x$ (con $a>0$, $a \neq 1$)	$\frac{1}{x \ln a}$
$\ln x$	$\frac{1}{x}$

Regole di derivazione

- $D(c) = 0$
- $D(cx) = c$
- $D[c\, f(x)] = c\, f'(x)$
- $D[f(x) + g(x)] = f'(x) + g'(x)$
- $D[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- $D\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$
- $D[f^{-1}(y_0)] = \frac{1}{f'(x_0)}$ con $y_0 = f(x_0)$
- $D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
- $D[f \circ g(x_0)] = f'[g(x_0)]\, g'(x_0)$

Regola di integrazione	Regola di derivazione
$\int k f(x)\,dx = k \int f(x)\,dx$	$D[k f(x)] = k f'(x)$
$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$	$D[f(x) + g(x)] = f'(x) + g'(x)$
$\int dx = \int 1\,dx = x + c$	$D(x) = 1$
$\int m\,dx = m \int dx = mx + c$	$D(mx + q) = m$
$\int x^{\alpha}\,dx = \dfrac{x^{\alpha+1}}{\alpha+1} + c \quad \alpha \neq -1$	$D(x^{\alpha}) = \alpha x^{\alpha-1}$
$\int a^{x}\,dx = \dfrac{a^{x}}{\ln a} + c \quad a>0,\ a \neq 1$	$D(a^{x}) = a^{x} \ln a$
$\int e^{x}\,dx = e^{x} + c$	$D(e^{x}) = e^{x}$
$\int \dfrac{1}{x}\,dx = \ln\lvert x\rvert + c \quad x \neq 0$	$D(\ln\lvert x\rvert) = \dfrac{1}{x}$
$\int [f(x)]^{\alpha} f'(x)\,dx = \dfrac{[f(x)]^{\alpha+1}}{\alpha+1} + c$	$D[f(x)]^{\alpha} = \alpha [f(x)]^{\alpha-1} f'(x)$
$\int e^{f(x)} f'(x)\,dx = e^{f(x)} + c$	$D\,e^{f(x)} = e^{f(x)} f'(x)$
$\int \dfrac{f'(x)}{f(x)}\,dx = \ln\lvert f(x)\rvert + c$	$D \ln\lvert f(x)\rvert = \dfrac{f'(x)}{f(x)}$